Introduction to MM_STD

This is the introduction file for the set of tutorials on Mike Day's notation for hand movements. It also effectively forms the first tutorial, or, perhaps more accurately, the zeroth tutorial.

These tutorials will go at a very gentle pace. Some of the concepts involved here are quite new, subtle and unusual. Please read carefully, do the exercises, and if you have any questions then ask. All feedback is useful, and you will be contributing to the development of this material. By giving us your feedback you will play a part in this contribution to juggling theory and practice.

Thank you for taking part.

C.D.Wright

Lesson 0 - What is this all about?

1. Existing notations do not cover hand movements.

As you may be aware, there is a notation for some juggling patterns called the SiteSwap notation. You don't need to know the SiteSwap notation in order to understand these tutorials, but they will occasionally be mentioned.

Actually, SiteSwaps will be mentioned quite a lot here in this introduction. If you don't know anything about SiteSwaps and feel that you're missing out, you can take a look at the SiteSwap information. For now, though, take our word for it that they are not essential.

One thing that the SiteSwap notation does not cover is hand movements. In particular, the SiteSwap for Mills Mess is just a simple "3". The thing that makes Mills Mess such a beautiful pattern is the smooth, fluid and hypnotic hand movements. SiteSwaps are used to describe the timing of a juggling pattern, but until quite recently there has been no simple way of writing down a description of the hand movements.

2. MM_STD does cover some patterns involving hand movements.

It was to solve this problem that Mike Day invented a system for describing hand movements as related to juggling. This was so successful that it then allowed new patterns to be developed and existing patterns to be analysed and modified. Just as SiteSwaps only

concentrate on one specific aspect of juggling, the timing, so the MM_STD concentrates on just one aspect, the hand movements.

Not every possible collection of hand movements is covered by this system. There are patterns that are not described by the MM_STD, although, as with SiteSwaps, there are extensions to the notation to cover most of these other patterns. As with SiteSwaps, the extensions are often technical and awkward, enhancing the notation at the cost of simplicity. We will concentrate in this series on the basic form of the notation, leaving the more extensive version for later.

3. Where does the name come from?

The full name of the notation is the *"Mills Mess State Transition Diagram."*

Don't you wish you hadn't asked?

The notation was originally designed to help analyse, understand and manipulate Mills Mess. We don't want to give away too much at this stage, so we won't say any more about what a "State Transition Diagram" actually is. That is the subject of the tutorial series.

4. How do I get more help?

If you want more help on any juggling subject then point your web browser at the Juggling Information Service at

```
http://www.juggling.org/
```

5. The next stage.

This is the end of the Introduction.

Go to Lesson 01

Dr C.D.Wright

Lesson 1 for MM_STD

Have you read the file Intro yet?? It forms the initial introduction to the tutorials, and is essential reading. As is said there, this series is going to be extremely gentle. You may be tempted to skim through a lesson and think that it's obvious, but be prepared to go back and re-read it. Some of the ideas are deep, and almost everything we write will be relevant, bordering on important. As the series goes on don't expect to be able simply to read each section. You may need to study rather than read. However, we believe that the material repays the effort.

So, Lesson 1.

We're going to assume you can juggle three balls and we'll start with a three ball cascade. We're also going to assume that you juggle with a long(ish) dwell time. Imagine having a ball in each hand and one in the air. As the one comes down, exchange it for the ball in the right hand, throwing fairly high. Now wait for it to come down and then exchange in the left.

Notice how the hands are full for most of the time.

We will take this as a basis for the discussion. Think of the hands being full all of the time, and that there is a succession of exchanges in alternate hands.

We're also going to assume that we're juggling to music, and that each exchange happens on a beat. Of course, an exchange needs both a throw and a catch, in that order, and there is a small time gap between them. Some people like to think of the throw as happening exactly on the beat and the catch slightly after, We like to think of the throw as anticipating the beat and the catch happens on the beat. It doesn't really matter. There is an exchange at every beat of the music.

Finally, we're only going to deal with patterns where the exchanges happen in alternate hands. An exchange in the left hand must be followed by an exchange in the right, and vice versa. We will see later how to relax this restriction.

So, in summary;

1. we juggle a 3 ball cascade with a long dwell time,
2. we juggle to music with an exchange on every beat,
3. the exchanges always alternate hands.

These basic assumptions form the foundation of the notation.

Go to Lesson 2
Back to Lesson 0

Dr C.D.Wright

Lesson 2 for MM_STD

Right, here is a reminder from Lesson 1;

1. we juggle a 3 ball cascade with a long dwell time,
2. we juggle to music with an exchange on every beat,
3. the exchanges always alternate hands.

Our final simplification that we're going to make is this. All throws and catches basically happen either with the arms uncrossed, with the arms crossed right over left, or with the arms crossed left over right. We're not going to be too concerned with any more precision about the places of throws and catches than this. Of course, in practice there is a much finer degree of control shown by jugglers than this, but we're going to pretend for now that the only thing that matters is the "crossedness" of the arms.

Now, let's juggle our ordinary three ball cascade and think about the right hand for a minute. When you do an ordinary inside throw the hand moves in a clockwise circle (from your point of view). It comes up on the inside for the throw and goes down on the outside for the catch. This is what we'll call an "Inside Exchange". In fact, whenever the right hand does a clockwise circle for the throw/catch exchange we'll call that an "Inside Exchange".

The same is true for the left hand, only for the left hand the "Inside Exchange" is characterised by an anti-clockwise circle.

The "Inside Exchange" is when you do a clockwise throw/catch circle with the right hand, or an anti-clockwise throw/catch circle with the left hand.

Warning: This definition will seem weird if the exchange happens on the "wrong" side of the body!

Let's do an "Inside Exchange" with the right hand, but let's do it on the left side of the body, with the arms crossed. The right hand still goes in a clockwise circle, because that's exactly what it means to say that you're doing an "Inside Exchange". Now, however, the hand is coming up on the outside, so you will notice that the path the ball will take will be the same as if it had been an outside throw with the left hand. Ignore this. We're not concerned for the moment about the actual path of the ball. We're only interested in which direction the hand goes round.

Of course, if the hand goes the *other* way round then we're going to call it an "Outside Exchange". This only seems fair and reasonable.

Summary

We're only going to worry about whether or not the arms are crossed, and if so, which is on top.

```
  Right hand goes      clockwise -> "Inside Exchange"
  Left  hand goes      clockwise -> "Outside Exchange"
  Right hand goes anti-clockwise -> "Outside Exchange"
  Left  hand goes anti-clockwise -> "Inside Exchange"
```

Go to Lesson 3
Back to Lesson 1

Dr C.D.Wright

Lesson 3 for MM_STD

Right, here is a reminder from Lesson 1;

1. we juggle a 3 ball cascade with a long dwell time,
2. we juggle to music with an exchange on every beat,
3. the exchanges always alternate hands.

Then in Lesson 2 we had:

```
1. We only worry about whether or not the arms
      are crossed, and if so, which is on top.

2. Right hand goes      clockwise -> "Inside Exchange"
3. Left  hand goes      clockwise -> "Outside Exchange"
4. Right hand goes anti-clockwise -> "Outside Exchange"
5. Left  hand goes anti-clockwise -> "Inside Exchange"
```

Because we are only concerned about whether or not the arms are crossed, and if they *are* crossed, which one is on top, there are only a few things to think about. In fact, let's consider a moment in our leisurely three ball cascade when we're holding two balls and waiting for the third ball to come down. Remember, because we have a long dwell time we spend most of the time with a ball in each hand waiting for one to come down for an exchange.

So here we are, a ball in each hand, about to do an exchange. A complete list of the things we are allowed to think about is this:

1. which hand is about to do the exchange,
2. which arm is on top, or are they uncrossed,
3. are we about to do an inside throw or an outside throw.

To save time and space, let's simply write the letter "r" or "l" to say which hand is to do the exchange, and let's write the letter "R", "U", or "L" to say which arm is on top. The "U" stands for "Uncrossed". We now have six possible "configurations" for our hands

```
Rl    Rr
Ul    Ur
Ll    Lr
```

Let's take one, say "Ur", and remind ourselves what it means. The "U" says that the arms are uncrossed, the "r" says that the right hand is about to do an exchange. What about "Rl"? That means that the arms are crossed with the right arm on top, and that the left hand is about to do an exchange.

We call these combinations "States", and they are absolutely fundamental to the notation. As we juggle and the hands move around, carrying balls or moving into positions for catches, so we move from one state to another.

Consider the false shower with all throws on juggler's right hand side. About to throw with the right hand, the arms are uncrossed, we are in state "Ur". What next? The next throw is with the left hand, of course (remember, the hands always alternate), and the throw is done with the arms crossed. We will therefore be in state "Rl" or "Ll", but which one?

Ah, well, there are in fact two types of false shower. When throwing with the left hand from the right side, it can be done either from over the top or from underneath the other arm. If the false shower is done with overarm throws then the pattern moves between the states

```
          Ur   <->   Ll
```
whereas if the left hand throw is done from underneath then the pattern moves between the states
```
          Ur   <->   Rl
```

Summary

Things are starting to take shape. What we have seen in this lesson is that some aspects of a juggling pattern can be described by looking at the different "States" the hands move through. There are six states,

```
            Rl     Rr
            Ul     Ur
            Ll     Lr
```

The letters stand for

```
  1.   "R"   ->   Right arm crossed over the top
  2.   "U"   ->   Arms uncrossed
  3.   "L"   ->   Left arm crossed over the top

  a.   "r"   ->   Right hand next to exchange
  b.   "l"   ->   Left hand next to exchange
```

These states refer to when the hands are both holding balls during the long dwell time, and when there is an exchange about to happen.

The next lesson looks at the different possible transitions between states.

Go to Lesson 4
Back to Lesson 2

Dr C.D.Wright

Lesson 4 for MM_STD

Well, here we are in lesson 4, and here is a quick recap on the main points of the previous lessons.

Lesson 1:

```
1. we juggle a 3 ball cascade with a long dwell time,
2. we juggle to music with an exchange on every beat,
3. the exchanges always alternate hands.
```

Lesson 2:

```
1. We only worry about whether or not the arms
      are crossed, and if so, which is on top.

2. Right hand goes      clockwise -> "Inside Exchange"
3. Left  hand goes      clockwise -> "Outside Exchange"
4. Right hand goes anti-clockwise -> "Outside Exchange"
5. Left  hand goes anti-clockwise -> "Inside Exchange"
```

Lesson 3:

```
There are six states the arms can be in.  These are
  written as  Rl, Rr, Ul, Ur, Ll, Lr.  The state we're
  in tells us which hand is next to do the exchange,
  whether the arms are crossed, and if so, which arm
  is on top.
```

Let's go back to the false shower with the throws happening on juggler's right. We have seen that there are two basic varieties, one with the left arm crossing on top, the other with the left hand crossing underneath. For the sake of simplicity let's concentrate on the left arm underneath variety. This means that the states we are in are ...

```
Ur    <--->    Rl
```

What kind of exchange is the right hand doing in the false shower? Well, from the juggler's point of view both hands are doing anti-clockwise circles. This means that the right hand is doing outside exchanges, but the left hand is doing inside exchanges. It's here that the care taken about the definition of inside versus outside exchanges pays off. So when we move from "Ur" to "Rl" it's with an outside exchange, but when we move back from "Rl" to "Ur" it's with an inside exchange. What we do then is to put markings on the arrows to show what kind of an exchange we do, like this ...

```
     .-----o----> Rl
Ur <----+-----'
```

We are, of course, up against the limitations of ASCII diagrams here, but you ought to know what we mean. The little "o" on the arrow means it's an outside exchange, the little "+" should be thought of a a bar across the arrow, which is an "i", selected to represent the inside exchange. This diagram represents the right-throw false shower, making outside exchanges with the right hand when the arms are uncrossed, and inside exchanges with the left hand when the left hand is crossed underneath. Ok, quiz time. Draw the diagrams for

1. the left-throw false shower with underneath throws,
2. the left-throw false shower with above throws,
3. the right-throw false shower with above throws,
4. the cascade,
5. the reverse cascade,
6. the right-throw half-shower.

Summary

When we move from one state to another we draw an arrow and say whether the exchange was an inside exchange or an outside exchange. In ASCII we represent an inside exchange with a "+", an outside exchange with an "o".

Go to Lesson 5
Back to Lesson 3

Dr C.D.Wright

Lesson 5 for MM_STD

Here's the main point again from Lesson 4:

```
When we move from one state to another we draw an arrow
  and say whether the exchange was an inside exchange or
  an outside exchange.  In ASCII we represent an inside
  exchange with a "+", an outside exchange with an "o".
```

Also recall the diagram we had for the right-hand throw half shower with underneath throws ...

```
                  .-----o----> Rl
                Ur <----+-----'
```

You are further supposed now to have drawn the diagrams for the following patterns ...

1. the left-throw false shower with underneath throws,
2. the left-throw false shower with above throws,
3. the right-throw false shower with above throws,
4. the cascade,
5. the reverse cascade,
6. the right-throw half-shower.

We're going to assume you've done this. If you haven't done these, take the time now. It will help quite a lot. For those of you who *have* done the exercises, here are the answers ...

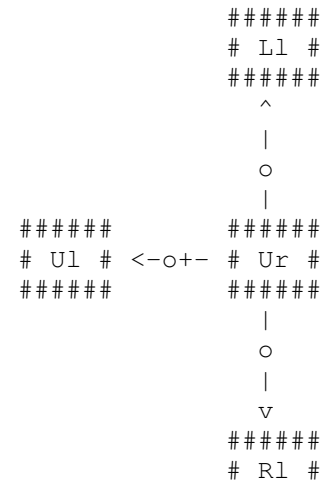
So, what now?

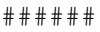
Suppose you're in the middle of juggling some pattern and you find yourself in the state "Ur". Remember, this means that your arms are uncrossed, and your right hand is about to do an exchange. What can we do now?

Looking at the diagrams above there are four things we've done so far from this situation. These are ...

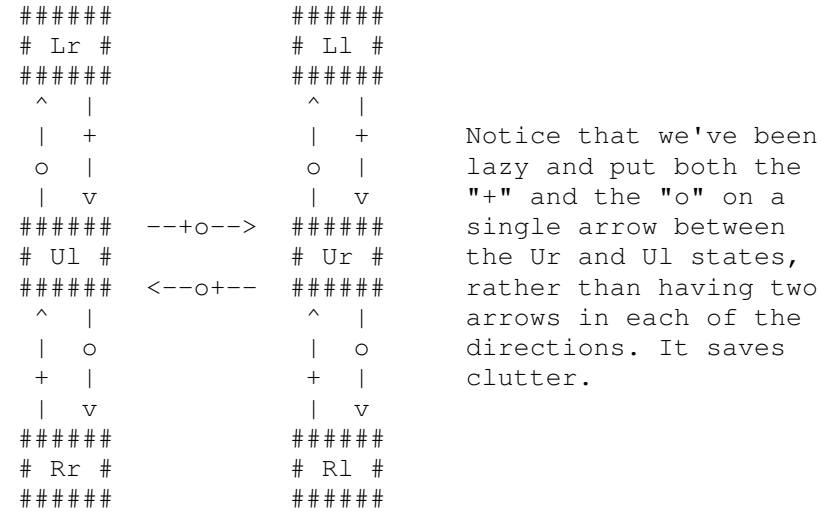
```
Ur --+-> Ul
Ur --o-> Ul
Ur --o-> Ll
Ur --o-> Rl
```

Notice that in each case the state we go to has a left throw next. Of course, this has to be the case because the hands always alternate. We can go to the state "Ul" in two different ways, one is with an inside exchange, the other is with an outside exchange. The other options are to move to an arms crossed position, either right on top or left on top, but in each case we have do do an outside throw. We can lump all this together in a single, two-dimensional diagram, like this ...





In fact, let's put in _all_ the states, and all the arrows we've seen so far ...



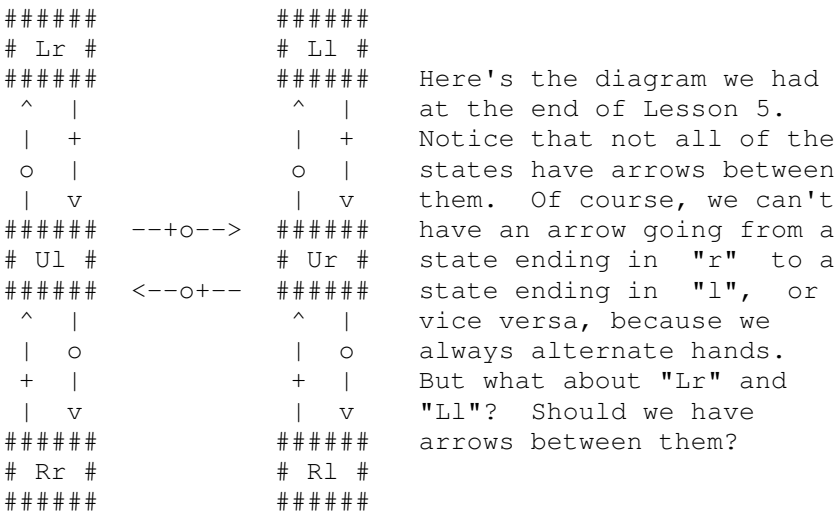
The basic idea now is that you can wander around on this diagram, doing an exchange every time you follow an arrow. If you follow an arrow marked with a "+" then you have to do an inside exchange, and if you follow an arrow marked with a "o" you have to do an outside exchange. Of course, if the arrow has both a "+" and a "o" then you can do either type of exchange.

This isn't the whole story. For one thing the diagram isn't yet complete. We also need to see how to use it to analyse patterns and invent new ones.

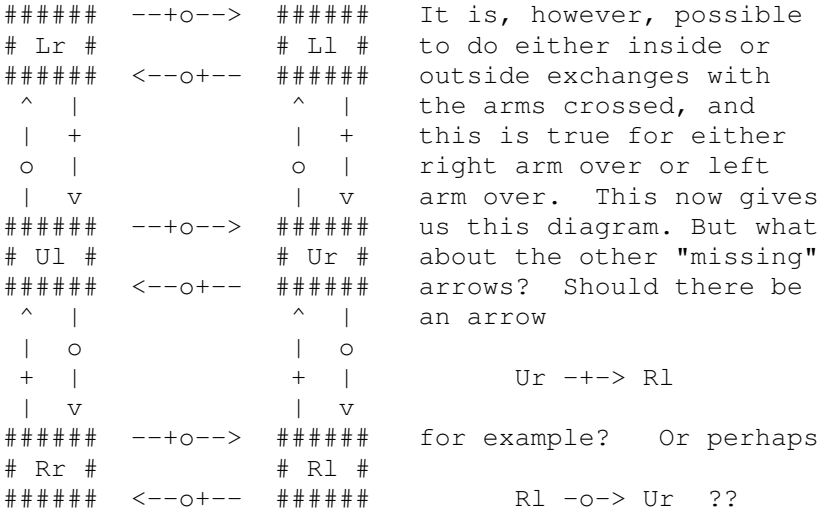
Go to Lesson 6
Back to Lesson 4

Dr C.D.Wright

Lesson 6 for MM_STD



The states Lr and Ll correspond to the arms being crossed, left over right, and if we are to have arrows between them then it would imply that you can juggle with your arms crossed. Well, of course you can. What about inside versus outside exchanges? When most people juggle with crossed arms they do inside exchanges, resulting in a pattern that looks like a reverse cascade. That may require some thought - it's not immediately obvious.



To answer this question, let's look closely at the possible arrow `Ur -+-> Rl`.

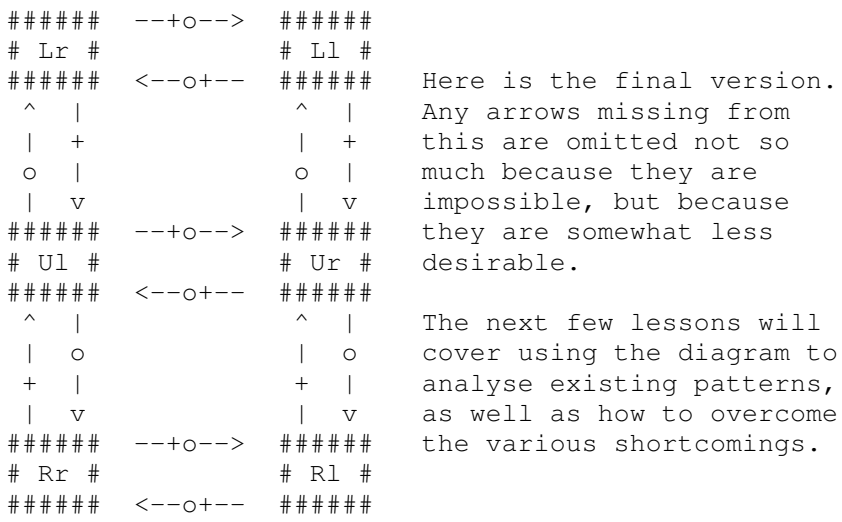
Firstly, this means that you start with the arms uncrossed and do an inside exchange with the right hand. This will happen more-or-less on the right hand side of the body, so the right hand throws from the centre(ish) of juggle space, moves further to the right, and catches. If this is done in the most fluid and natural way, this results in the arms being even more uncrossed (if that makes any sense) than when they started. In particular, the most fluid and natural movements certainly do not result in the arms being crossed. This possible transition is therefore not included in the diagram.

The same is true of all the following arrows.

```
Ur -+-> Rl              Rl -o-> Ur
Ur -+-> Ll              Ll -o-> Ur
```

These arrows are all excluded from the diagram because the movements they represent do not feel natural. This does not mean that they are impossible or uninteresting. It just means that they are, in some sense, for the moment, less desirable.

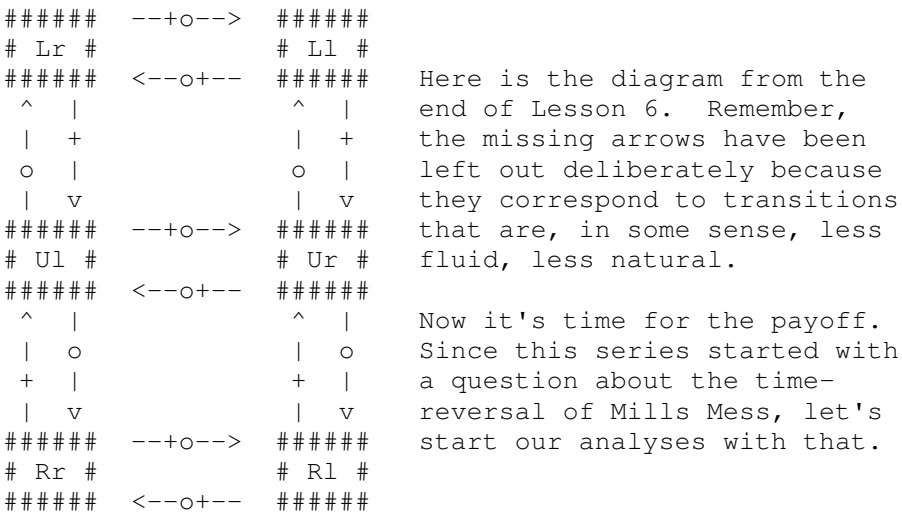
Finally, what about the potential arrows Lr <--> Rl and Rr <--> Ll ?? These transitions require an almost instant change from having the arms crossed over one way, and then crossed the other way. Again, this is not impossible, but the movements involved are substantial and, as before, lack the fluidity and naturalness of the other transitions.



Go to Lesson 7
Back to Lesson 5

Dr C.D.Wright

Lesson 7 for MM_STD



We need to start by working out what Mills Mess is on the diagram. On the way we'll see some of the limitations of the diagram, as well as the basic power of the technique.

Mills Mess. I'll start the description and analysis from where you have the left arm crossed over the right, and you're about to do an exchange with the left hand. This throw therefore comes from the right side. Here's a full description of the sequence of throws:

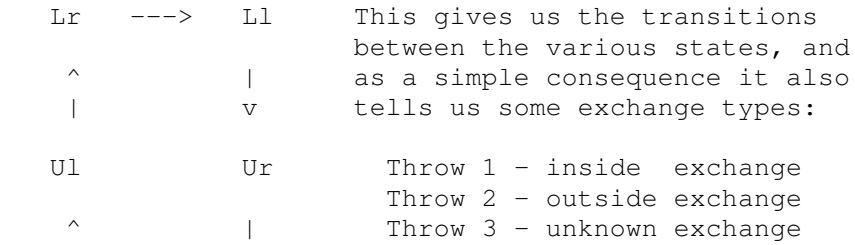
	Hand	Side	Arms
1.	Left	Right	Crossed - left on top
2.	Right	Right	Uncrossed
3.	Left	Right	Crossed - right on top
4.	Right	Left	Crossed - right on top
5.	Left	Left	Uncrossed
6.	Right	Left	Crossed - left on top

All catches are made more-or-less in the middle of juggle space. Remember that, and we'll come back to it later.

So, what does all this mean on the diagram? We can easily write down what the states are, so let's add them to the chart above ...

	Hand	Side	State	Arms
1.	Left	Right	Ll	Crossed - left on top
2.	Right	Right	Ur	Uncrossed
3.	Left	Right	Rl	Crossed - right on top
4.	Right	Left	Rr	Crossed - right on top
5.	Left	Left	Ul	Uncrossed
6.	Right	Left	Lr	Crossed - left on top

So this gives us the (cut down) diagram as follows:



```
 |          v        Throw 4 - inside  exchange
                     Throw 5 - outside exchange
Rr   <---   Rl       Throw 6 - unknown exchange
```

We know what type of exchanges occur on throws 1, 2, 4 and 5 because only one type is permitted by the state transition diagram. But what about the unknown exchanges?

Throw 3 is with the left hand from the right side of the body and the catch is in the middle of juggle space. This means that between the throw and catch of the exchange the hand moves to the left. That makes it an inside exchange. Let's put that in, along with all the other transition types, just for the sake of completeness.

```
Lr   -+->   Ll     So what was the point above about
                   the catches?
 ^          |
 o          +      Well, at the very beginning we
 |          v      assumed that the exchange happened
                   more-or-less in a single location.
Ul          Ur     This isn't true in Mills Mess. When
                   going from  "Ll"  to  "Ur"  with our
 ^          |      inside exchange, the catch happens
 +          o      when the arms have uncrossed.  This
 |          v      contrasts with the assumption that
                   the throw and catch happen, and then
Rr   <-+-   Rl     the hands move to their new places.
```

However, the notation doesn't really cover the actual hand positions. This is one of its major limitations, although there are ways of getting around it. In fact, when you're juggling you tend not to have extremely long dwell times. This means that, in general, the catch happens close to the position specified by the _next_ state, not this one.

A specific example is the Ll -+-> Ur transition. If we did have the long dwell time then the catch would happen with the arms still crossed. However, for Mills Mess the catch happens in the uncrossed position, as specified by the next state.

We will assume from here on that this is, in fact, the case. We'll assume that the catch happens close to the position specified by the next state.

So, Mills Mess is a clockwise traversal of the diagram, following the "inside exchange" arrows whenever you have a choice. And the time reversal ?? That's the subject of the next lesson.

Go to Lesson 8
Back to Lesson 6

Dr C.D.Wright

Lesson 8 for MM_STD

```
Lr   -+->   Ll

 ^          |      So here is the transition diagram
 o          +      for Mills Mess.  We go around the
 |          v      full version of the diagram in a
                   clockwise direction, and we do
Ul          Ur     inside exchanges whenever we have
                   a choice.
 ^          |
 +          o
 |          v      So what is the time reversal?

Rr   <-+-   Rl
```

In order to see what it means to "time-reverse" things let's start by concentrating on just one transition. Specifically, let's look at the time-reversal of the transition ...

```
   Ll -+-> Ur
```

Broken down into its constituent parts, this means ...

```
( 0. you've just caught with the left hand, )
  1. the arms are crossed, left on top,
  2. throw with the left hand,
  3. move the left hand to the left,
  4  uncross the arms from left on top,
  5. catch with the left hand, (arms uncrossed)
( 6. get ready to throw with the right hand. )
```

Take a moment here to marvel at how concise the transition notation is, comparing "Ll -+-> Ur" with the verbosity of the above list of actions.

To time reverse, spatial directions must change, every throw becomes catches and catches become throws, and the sequence of events reverses. Writing this in full gives us ...

```
( 6. you have just CAUGHT with the right hand. )
  5. THROW with the left hand, (arms uncrossed)
  4  cross the arms, left on top,
  3. move the left hand to the RIGHT,
  2. CATCH with the left hand,
  1. the arms are crossed, left on top,
( 0. get ready to THROW with the left hand, )
```

We have retained the original numbering here to help the matching the new version with the old, and anything that's changed (apart from the ordering, of course) has been put in upper case. Take some time to convince yourself that this is an accurate reversal of what happens.

So, what does this translate to on the transition diagram? The arms must start uncrossed, and the left hand throws, so we start with "Ul". After throwing, the left hand moves to the right, so the exchange is an outside exchange. Finally, the arms finish crossed, with the left arm on top. The resulting state is therefore "Lr". Thus we have the time- reverse transition

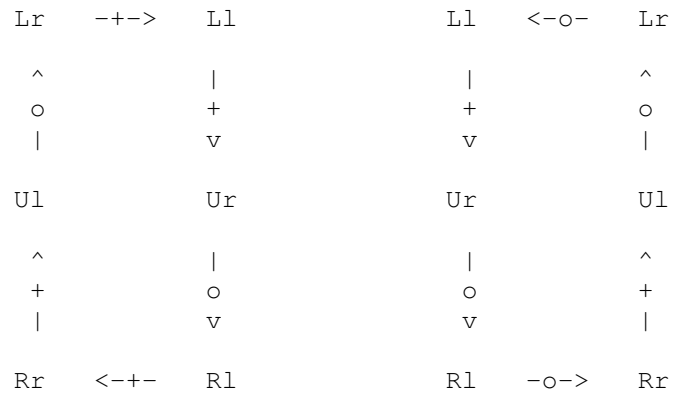
```
   Lr <-o- Ul
```

Careful analysis of the above reasoning shows that time- reversal is achieved as follows:

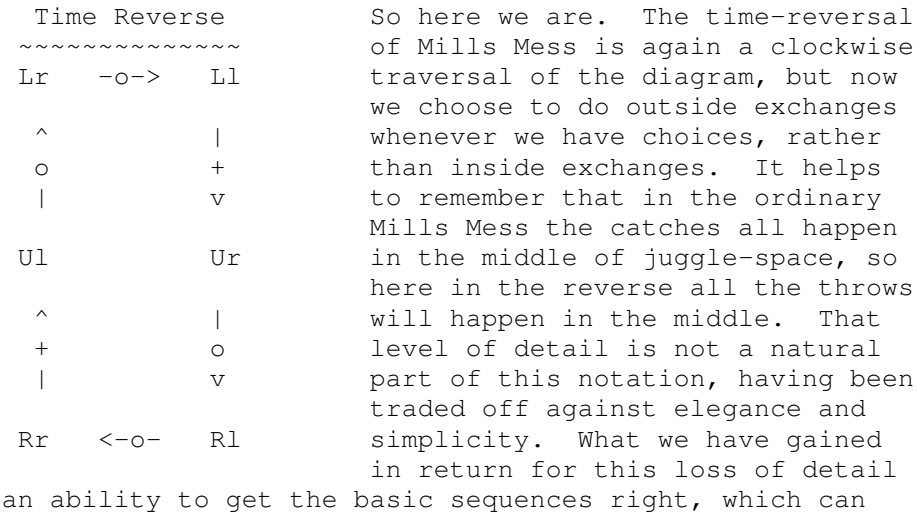
```
1. reverse the arrow direction,
2. swap inside and outside exchanges,
3. swap which hand is next to throw.
```

What does this imply for Mills Mess? Well, here is the before and after picture.

```
   Mills Mess              Time Reverse
~~~~~~~~~~~~~~          ~~~~~~~~~~~~~~
```



The only problem is that the states are no longer listed in the order we're used to. Because we have changed the hand that is about to throw, each state has "swapped sides", as it were. "Lr" is not top right instead of top left on the diagram. We can put them back in their usual places simply by swapping the diagram right to left. Note that the arrows at the top and bottom have to be reversed. The result is shown below.



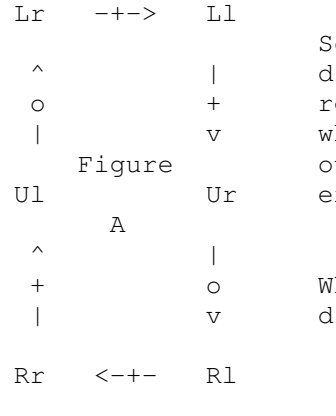
So here we are. The time-reversal of Mills Mess is again a clockwise traversal of the diagram, but now we choose to do outside exchanges whenever we have choices, rather than inside exchanges. It helps to remember that in the ordinary Mills Mess the catches all happen in the middle of juggle-space, so here in the reverse all the throws will happen in the middle. That level of detail is not a natural part of this notation, having been traded off against elegance and simplicity. What we have gained in return for this loss of detail is an ability to get the basic sequences right, which can then be adjusted to get the exact positioning later.

We have now accomplished our initial goal. We have seen the development of the state transition diagram that Mike Day invented to help him analyse Mills Mess. In the next few lessons we'll explore just some of the possibilities that this new notation opens up for us.

Go to Lesson 9
Back to Lesson 7

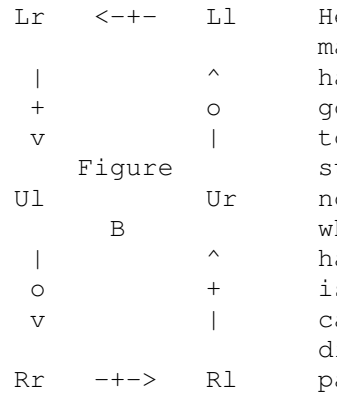
Dr C.D.Wright

Lesson 9 for MM_STD

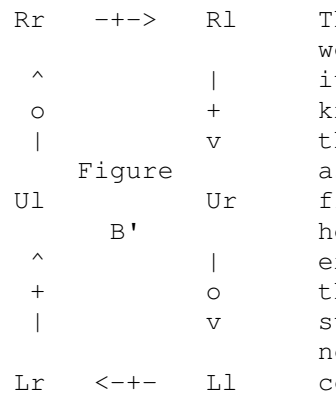


So here once again is the transition diagram for Mills Mess. The time reversal is the same, except that wherever we have a choice we do outside exchanges rather than inside exchange.

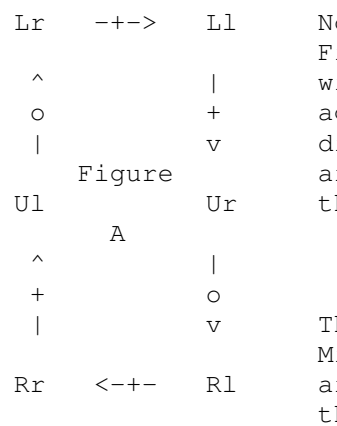
What do we get if we go around the diagram in the other direction?



Here is the direction reversal, making inside throws wherever we have a choice. Of course, you can go through the diagram laboriously to work out what happens at each stage, and we recommend you do this now both to make sure you understand what the diagram means, and so you have some idea of what the pattern is. However, we'll assume that you can do that and we'll show you a different way of analysing the pattern.



This new method of analysis asks how we can change this diagram to make it look like something we already know? Since the pattern goes around the diagram in the wrong direction, as it were, one thing we can try is flipping it top for bottom. Shown here at left is the result of doing exactly that. States that were at the top are now at the bottom, states that were at the bottom are now at the top. Take a moment to convince yourself that this is true.



Now compare the previous diagram, Figure B', the reverse traversal, with the Mills Mess Figure, shown again here at left. What is the difference between them? The arrows are all the same, the same hand is throwing in each case, ...

The wrong arm is on top!

This pattern is exactly the same as Mills Mess, except that whenever the arms are crossed, they are crossed the opposite way over to usual.

This is the pattern referred to as the "Funky Mess". The word "Funky" is used to describe a pattern that has been created by throwing under arm instead of over, and over arm instead of under. We can see it now as simply reversing the direction of the path in the diagram.

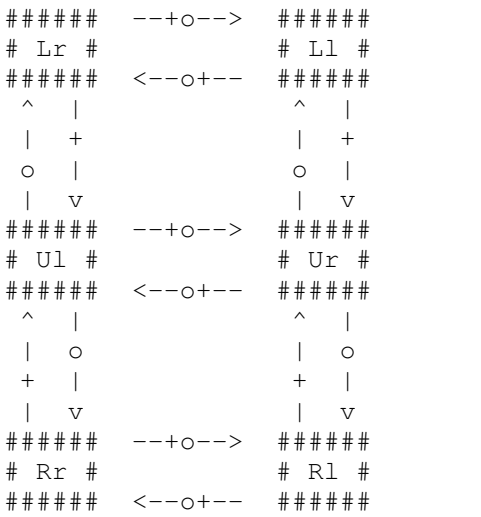
Again, as with Mills Mess, the time reversal is exactly the same, but following the "Outside Exchange" arrows instead of the "Inside Exchange" arrows. This gives us a total of four basic patterns; Mills Mess and its time reversal, and the Funky Mills Mess and its time reversal. More than that, we can see from the diagram that unless you start mixing up the inside and outside exchanges (where you have choices) in the same pattern, these are the only four possibilities.

In the next lesson we'll look at a couple of other patterns and see how they can be described on the diagram.

Go to Lesson 10
Back to Lesson 8

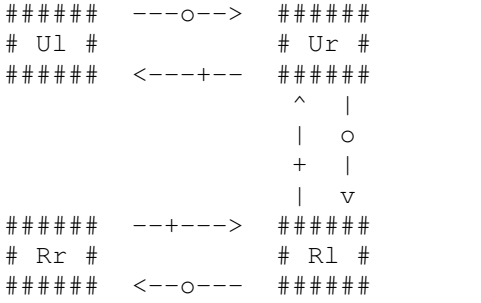
Dr C.D.Wright

Lesson 10 for MM_STD



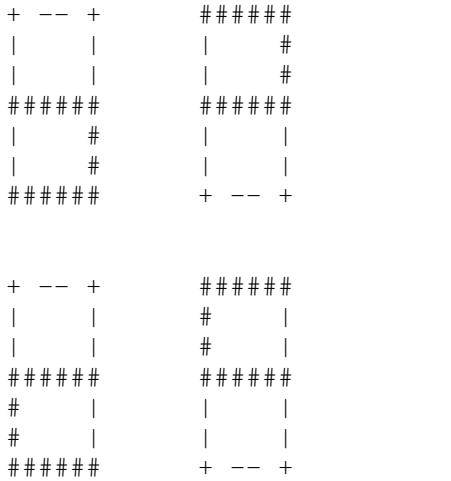
So here is our diagram again, and we have used it to analyse Mills Mess and compute its time-reversal. We have also used it to see what the "Funky Mess" is, and how it's a fairly natural idea.

But what about some other patterns?



Here's another pattern. In this pattern we see-saw back and forth between the states Ul and Rr, going via Ur and Rl on the way. Firstly, can you work out what this pattern is? Interpreting these diagrams is never easy, so take some time to try it.

Here is the answer to the pattern above.



Drawn at left here are four "skeletons" of the state transition diagram. The first has highlighted on it the pattern above - the three-cylinder-engine. This particular version has the balls being throw on the right, then middle, then the left, and has the right arm crossing on top. The other three give the alternatives - having the other arm on top, throwing the balls in the opposite direction, or both.

Transitions between these different forms of the patterns can easily be worked out by following the paths around on the full diagram, but actually juggling them may be more of a challenge !! As with all useful notations, the gain in understanding often (initially) outstrips the ability to put it into practice. However, with any luck the notation will eventually help us develop more flexibility in our juggling by giving us things to try that we'd never thought of, and the ability to understand them and communicate them to others unambiguously.

In the next lesson we'll see how to combine the diagram with SiteSwaps, and how we can sometimes avoid having to draw the full diagram.

Go to Lesson 11
Back to Lesson 9

Dr C.D.Wright

Lesson 11 for MM_STD

We have now seen how to write clear, concise and unambiguous descriptions of hand movements by using the state transition diagram invented by Mike Day, and how to use the notation to analyse patterns, working out the time reversal and related patterns.

In this lesson we'll see how to combine this hand movement notation with the better known SiteSwaps. Of course, this may be a bit of a problem if you're not already familiar with SiteSwaps, but you don't need to worry. SiteSwaps will be explained as we go along, so you can pick up those bits that are necessary. If you decide after reading this lesson that want to know more about Siteswaps, contact us and we'll send you a gentle introduction to them.

In the meantime, let's get on with seeing how they can help us write descriptions of other patterns. We'll start by giving a quick overview of SiteSwaps. If you already know all about SiteSwaps then feel free to skip the boxed section below, although it can't do you any harm just to make sure that we've got it right!

```
+-------------------------------------------------------------+
|                                                             |
| Basic Siteswaps ...                                         |
|                                                             |
|    1. only describe patterns with a fixed, even rhythm,     |
|    2. only describe patterns that alternate hands,          |
|    3. don't describe multiplex or synchronous patterns,     |
|    4. don't describe hand movements or exchange types,      |
|    5. only describe patterns with exactly two hands.        |
|                                                             |
| In SiteSwaps, consecutive throws are described by a sequence |
| of numbers, and these numbers are then interpreted to give a |
| juggling pattern.  Not all sequences correspond to juggling  |
| patterns, but we we won't go into the how and why of the     |
| sequences here.  All we'll do is mention how to interpret    |
| the numbers, because that's enough for our purposes here.    |
|                                                             |
|   1. For any number from 3 onwards ...                      |
|      Imagine juggling that many balls in either a cascade    |
|      or fountain.  The kind of throw you are now imagining   |
|      is the kind of throw this number represents.           |
|                                                             |
|   2. For the number 2 ...                                   |
|      Imagine holding a ball for one beat.                   |
|                                                             |
|   3. For the number 1 ...                                   |
|      Imagine transferring a ball from one hand to the        |
|      other.  It's also known as a "shower throw."           |
|                                                             |
|   4. For the number 0 ...                                   |
|      This is having an empty hand for one beat.             |
|                                                             |
| Here are some example, all done with three balls and        |
| "patched in" to a three ball cascade.                       |
|                                                             |
| ... 3 3 3 4 2 3 3 3 ...                                     |
|           ^^^          Juggle a three ball cascade. Then do |
|                        a fountain-like throw from one hand  |
|                        and pause with the other for a beat. |
|                        You can wave that ball around if you |
```

```
|                          like, but you don't have a lot of    |
|                          time.                                |
|                                                               |
| ... 3 3 3 5 2 2 3 3 3 ...                                     |
|           ^^^^^          Juggle a three ball cascade. Then do |
|                          a single high throw and wait for it  |
|                          to come down. If you get the height  |
|                          of the high throw right you'll have  |
|                          a wait of exactly two beats.         |
|                                                               |
| ... 3 3 3 5 5 5 0 0 3 3 3 ...                                 |
|           ^^^^^^^^^      Start with a three ball cascade. Do  |
|                          three high throws, leaving you with  |
|                          two empty hands. When the balls all  |
|                          come down you can resume your        |
|                          cascade.                             |
|                                                               |
| ... 3 3 3 4 4 1 3 3 3 ...                                     |
|           ^^^^^          Start with a three ball cascade.     |
|                          Then from one hand do a fountain-    |
|                          like throw, and immediately from the |
|                          other hand do the same. You have one |
|                          ball left, so transfer it across.    |
|                          Strangely, you can now carry on with |
|                          the cascade without breaking rhythm. |
|                                                               |
+---------------------------------------------------------------+
```

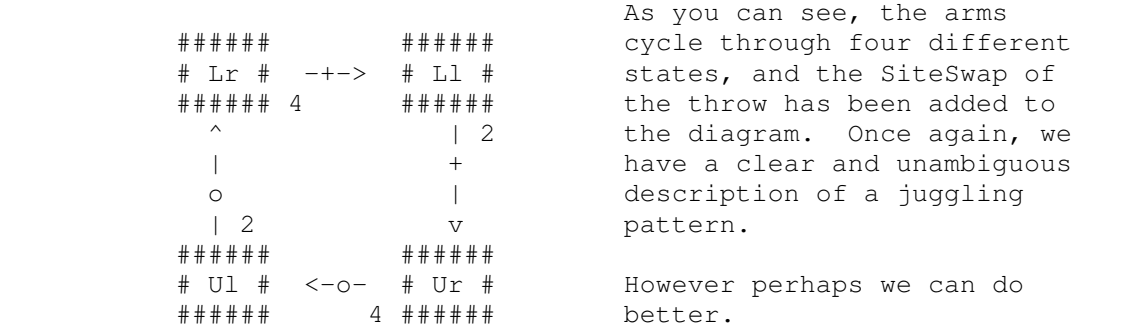
OK, that's the overview of SiteSwaps. How can we combine them with the state transition diagram? Let's look at a specific example, the weave. One hand, say, the left, holds a ball and never throws it. The other hand, the right hand, does two balls in columns. The SiteSwap for this is simply "42", one hand always throwing balls back to itself, the other hand always holding a ball. What about the hand movements?

Well, here is the full analysis of the pattern. Again, take time to check it, firstly for your own understanding, but also because we may have made a mistake!

Here is the sequence of states with the SiteSwap of the throw underneath ...

```
... -+-> Ur -o-> Ul -o-> Lr -+-> Ll -+-> Ur -o-> ...
         4       2       4       2       4
```

And here is the diagram.



Our basic idea here has been to add to the diagram the SiteSwap value of the throw being made. The main problem we're starting to run into is the fact that we have to write down the entire diagram each time, and adding things to it makes it just that little bit more awkward. Can we come up with something a little easier to deal with, easier to write down, easier to interpret, and less space-consuming?

Yes we can, and that's the subject of the next lesson.

Go to Lesson 12
Back to Lesson 10

Dr C.D.Wright

Lesson 12 for MM_STD

Let's have another look at the weave,

```
           +-- These are the same states --+
           |                               |
           |                               |
  ... -+-> Ur -o-> Ul -o-> Lr -+-> Ll -+-> Ur -o-> ...
           4       2       4       2       4
```

If we assume that we start with a right hand throw then we can ignore the second letter in each state. We know that we have to throw with alternate hands, so we don't have to put that in the states. Here's a new version, still with the letters for the hands, but moved, and put in brackets ...

```
          (r)     (l)     (r)     (l)     (r)
  ... -+-> U -o-> U -o-> L -+-> L -+-> U -o-> ...
           4       2       4       2       4
```

What next? Suppose we throw the ball and then cross the arms. It isn't really so much that we need to know which arm is on top, it's more interesting to know whether the throwing arm is going to cross ABOVE or BELOW the other arm. This is a bit of a shift in thinking, but it does release us completely from having to remember left and right. It lets us think about the throwing hand/arm, rather than having to think about absolute left or right. If we add this to our pattern above, (remembering that we started with the right hand,) we get something like this ...

```
          (r)     (l)     (r)     (l)     (r)
  ... -+-> U -o-> U -o-> L -+-> L -+-> U -o-> ...
           4       2  A    4  B    2       4
```

We perform the first "2" with the left hand, and then the left arm crosses on top. We can think of that as doing the throw and continuing the hand movement to cross above. In a similar way, the following "4" ends with the throwing hand, (the right) crossed below. We can now omit L, R and U entirely from our description, giving us ...

```
  ... -+-> . -o-> . -o-> . -+-> . -+-> . -o-> ...
           4       2  A    4  B    2       4
```

and we can rearrange this slightly to give this ...

```
  ... -+-> 4 -o-> 2A -o-> 4B -+-> 2 -+-> 4 -o-> ...
```

What remains? Well, since SiteSwaps are usually thought of as having only inside exchanges, we can make it a convention that unless we say otherwise, every exchange is an inside exchange. What about outside exchanges?

Mike Day's convention is to put a bar over any outside throw.

This means the ordinary cascade is "3", the reverse cascade is $\overline{3}$, the three ball half shower is $3\overline{3}$ and the four ball half shower is $\overline{53}$.

Using this idea we make almost our last change to the notation. Our running example, the weave, can now be written as ...

$$\overline{4}\ \overline{2}\ 4\ 2$$
$$\quad A\ B$$

No bar above a number means an inside exchange, a bar means an outside exchange. No subscript on a number means the hands finish uncrossed, an "A" means the throwing hand finishes above the other arm, a "B" means the throwing hand finishes below the other arm.

Our last remaining notational problem is having to place the bars above the numbers and the A's and B's below. We can "fix" this (at a cost of some readability) by putting "i" or "o" after _every_ number, and putting "A", "B" or "U" after every number, so the weave finally ends up as ...

```
4oU 2oA 4iB 2iU
```

This is, perhaps, slightly less readable than the version with bars and subscripts, but it does allow us to put everything in a single line. More, it is simply a way of writing down what has so far been done on the diagram. You will have to form your own opinion as to which you prefer, but we'll use the last of these in our next lesson.

Go to Lesson 13
Back to Lesson 11

Dr C.D.Wright

Lesson 13 for MM_STD

Now we have a notation for both SiteSwaps and hand movements combined. We have seen how Mills Mess can be annotated in the system, and how it can be analysed to compute its time reversal, as well as finding patterns that are related in other ways, in particular, we saw the Funky Mess, and lots of varieties of the "Three-cylinder-engine." The notation doesn't tell us everything, though. In particular, the description of the weave doesn't say that the arms should be at about the same level. it allows the situation where one ball is waved about completely above the other balls, even though this is, perhaps, a rather silly pattern. (Mind you, the public tends to like stupid patterns!)

The analysis for time reversal is more difficult when we include the SiteSwaps, but that's to be expected because the time reversal of SiteSwaps themselves isn't completely obvious.

However, at this point we can give the notation for several well-known patterns, including those we've seen already. Our thanks to Mike Day for these.

```
Mills Mess 3:          3iU 3oA 3iB
Mills Mess 4:          4iU 4oA 4iB
Mills Mess 42:         4iU 2oA 4iB 2iU 4oA 2iB
Mills Mess 534:        5iU 3oA 4iB
                  or   3iU 4oA 5iB
                  or   4iU 5oA 3iB
Mike's Mess:           5iB 2iU 2oA
Ssem Sllim             3iU 3oA 3oB  (time reversed MM)
Funky Mills Mess 3:    3iU 3oB 3iA
2-in-1 weave:          4oU 2oA 4iB 2iU
Alternating weave:     4oU 2oA 5iB 2iU 2iU
Burke's Barrage:       4iB 2iU 3oA         ie MM  2iU 3iU 4iU
Rubenstein's Revenge:  5iB 2iU 2oA 3iU 3oA
Egnever S'nietsnebur:  5oB 2iU 2oA 3oB 3iU  (time-reversed RR)
Sean's Sequel:         4iB 2iU 3oA 3iU 3oA
```

Sean's Sequel is a pattern discovered by Sean Gandini when learning Rubenstein's Revenge.

Here are a few of these in the bar and subscript notation. As we said earlier, some people will find these easier to read, but they are harder to write in plain ASCII, having to worry about the multi-line nature of them.

```
Mills Mess 3:                  33 3
                                 A B

Burke's Barrage:               4 23    ie MM 234
                                B  A

2-in-1 weave:                  42 4 2
                                 A B

Rubenstein's Revenge:          5 22 33
                                B  A  A

Sean's Sequel:                 4 23 33
```

```
B   A   A
```

The notation given for Rubenstein's Revenge needs a little explaining. It's not exact the way it's written, but needs a small amount of extra text. We need to say that the timing is slightly distorted from what's given, the 5 has to be low, the 2's have to be rushed a little. Also, one of the catches has to be a claw, although in the heat of doing it at speed this comes naturally. But perhaps the most amazing thing is that the notation can do Rubenstein's Revenge at all.

Go to Lesson 14
Back to Lesson 12

Dr C.D.Wright

Lesson 14 for MM_STD

We have seen in Lesson 8 how to compute the time-reversal of an ordinary pattern. The question now is, how do we compute the time-reversal of a SiteSwap with hand movements?

This is a good question!

We'll start with a comparatively simple answer, and then go from there. What follows here first is a purely mechanical procedure for computing the time-reversal of a pattern. We will use Rubenstein's Revenge as our example.

```
  Rubenstein's Revenge:  5iB 2iU 2oA 3iU 3oA
```

First, write it all in reverse order ...

```
  Reverse order ...   :  3oA 3iU 2oA 2iU 5iB
```

Next, change all "i"s to "o"s and vice versa and change all "A"s to "B"s and vice versa

```
             From  ...   3oA 3iU 2oA 2iU 5iB
             to    ...   3iB 3oU 2iB 2oU 5oA
```

Now move all "A"s, "B"s and "U"s one place earlier, moving the first one, which would sort of fall off the front, to the last place ...

```
             From  ...   3iB 3oU 2iB 2oU 5oA
             to    ...   3iU 3oB 2iU 2oA 5oB
```

So far this is exactly the same as our previous reversing procedure, except that it's being done in the new notation rather than on the diagram. The last few steps take into account the SiteSwap we have.

Move each number two places right, moving the ones that fall off the end back on at the front ...

```
             From  ...   3iU 3oB 2iU 2oA 5oB
             to    ...   2iU 5oB 3iU 3oA 2oB
```

And for now the last step. Take each number and move it backwards. The amount by which to move it is given by the number itself. For example, move a 2 backwards by two places, and move a 3 backwards three places. When you fall off the front, come back on at the end. This will mean that the 5 will stay where it is.

```
             From  ...   2iU 5oB 3iU 3oA 2oB
             to    ...   3iU 5oB 2iU 2oA 3oB
```

That's it !!

You may wonder if and why this last bit always works. After all, since the numbers move by different amounts, why are we never unlucky and end up with two numbers being moved to the same place? Well, it turns out that if you do end up with that sort of clash, then the original sequence of numbers can't have been a valid SiteSwap. This is _precisely_ the requirement for a sequence to be a valid SiteSwap, so if you start with a valid SiteSwap, it never goes wrong.

Well, of course, we've given little or no justification for most of this, and you might wonder where it all came from. As we said above, the origins of the first few steps can be traced back to the time-reversal on the diagram. The last two steps are derived from the time-reversal of an ordinary SiteSwap. To see the comparison, here is that process applied to the SiteSwap "1 2 3 4 5" ...

First, write it backwards ...

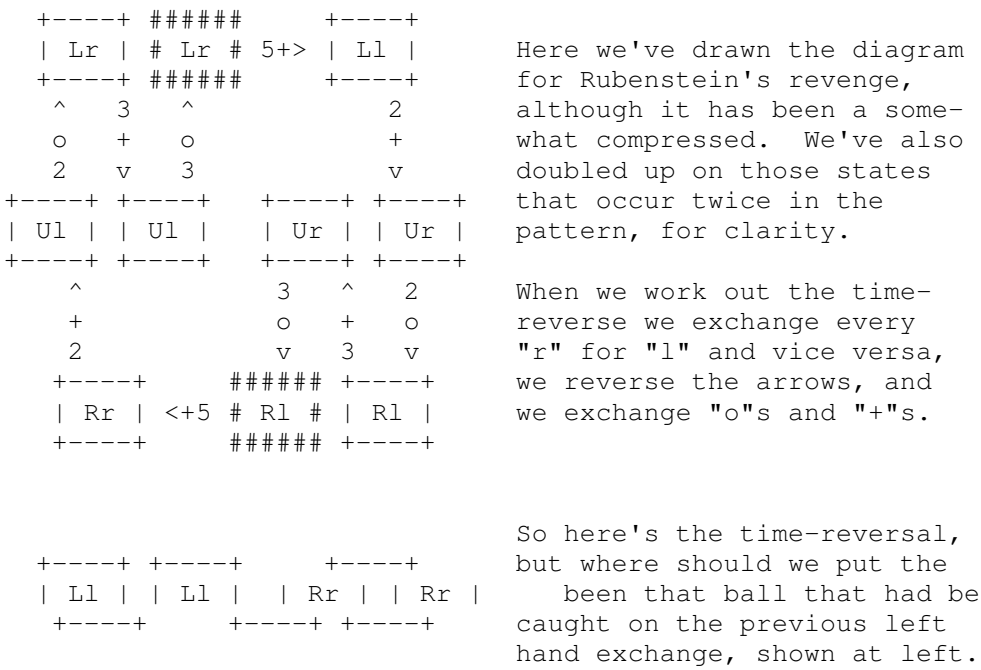
```
                5 4 3 2 1
```

Now, from each place count backwards by the amount given by the SiteSwap value. For example, from the 2, count backwards by two.

```
From  ...    5   4   3   2   1
to    ...    5   2   4   1   3
```

That's it. You can see the similarity in the processes. The main difference is that in the full process above we shift the numbers by two places. A shift like that is irrelevant in ordinary SiteSwaps, but here it serves to make sure that the SiteSwap values are lined up correctly with the arm movements.

We can see the whole process in action on the full diagram.



So, the "5" thrown in the normal-time pattern from the state "Lr" becomes a "5" thrown from the marked state above in the time-reversal.

So that's how we compute the time-reversal of a SiteSwap with hand movements, and it brings us close to the end of the tutorial series. In our next lesson we'll review some of what we've seen, and take a second look at some of the details and terminology.

Go to Lesson 15
Back to Lesson 13

Dr C.D.Wright

Lesson 15 for MM_STD

So, the time has come to review what we have seen, and to re-think some of our ideas. In particular, it's time to have a look at what the notation _can't_ do.

Let's look for a moment at four ball Mills Mess. There is a common mistake made in this pattern. When throwing the underarm throw, the ball should follow the previous throw, arcing up from the side and following more-or-less the same path through the air. However, quite a lot of people throw that underarm throw from a somewhat narrower position, and as a result it appears "inside" the previous ball. As we said above, this is a common mistake, and it's one that the notation as described does not help us with.

Similarly, as mentioned in Lesson 13 with regards the weave, the notation doesn't tell us that the held ball should weave between the others. It could equally well be waved about entirely above the other balls.

For a more complete description of a pattern, one can turn to the extension of SiteSwap used in JuggleKrazy. In this notation the spatial positions of each throw and catch can be specified, as well as any shifts in the timing. Because of this, the notation can easily be used to differentiate between the two versions of 4-ball Mills Mess, or the two versions of the weave described above. For more details on this notation please feel free either to ask us directly, or to download the free, interactive demonstration version of JuggleKrazy, which comes with full on-line help on the notation (and just about everything else!)

Still, the notation we've been working on here, despite its short-comings, is a very powerful tool, both for describing patterns, and for working out new ones.

However, there are are few minor quibbles that some people have, and which are now worth asking you all about.

```
1. The name
===========
     Although originally used to analyse Mills Mess,
   there is a feeling that the name shouldn't really
   contain the term "Mills Mess".  Also, calling it
   the State Transition Diagram doesn't really help
   its image with the general juggling community.
   Suggestions for a new name, anyone?
   Suggestions include

       MDHMD - Mike Day's Hand Movement Diagram
       MDHMN - Mike Day's Hand Movement Notation

   for the diagram and the text form respectively.


2. "Inside" and "Outside" exchanges
===================================
     A few of you have had trouble with the terms
   "Inside" and "Outside".  Certainly the terms are
   considered obvious by some, while others think
   they are completely unnatural.  Suggestions are
   welcome for alternative names for these.  Don't
   forget, though, that new suggestions should be
```

```
    accompanied by corresponding suggestions for the
    symbols on the diagram !!


3. Diagram symbols
==================
      Even if we _do_ keep the terms  "inside"  and
    "outside",  the "+" and "o" used on the diagram
    are not universally acclaimed.  Suggestions for
    alternative symbols are welcome.
```

Any and all suggestions welcome. Also, if anyone has any comments on the lessons themselves we'd be very grateful. Grammar, punctuation, detail, overview, motivation, all are fit subjects for comment.

Please, feedback requested !!

C.D.Wright

Back to Lesson 14

Dr C.D.Wright